# Physics from information

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This is an ongoing review on the idea that the phase space information loss at causal horizons is the key ingredient of major physical laws. Assuming that information is fundamental and the information propagates with finite velocity, one can find that basic physical laws such as Newton's second law and Einstein's equation simply describe the energy-information relation (dE=TdS) for matter or space time crossing causal horizons. Quantum mechanics is related to the phase space information loss of matter crossing the Rindler horizon, which explains why superluminal communication is impossible even with quantum entanglement. This approach also explains the origin of Jacobson's thermodynamic formalism of Einstein gravity and Verlinde's entropic gravity. When applied to a cosmic causal horizon, the conjecture can reproduce the observed dark energy and the zero cosmological constant. Quantum entanglement, path integral, and holography are also natural consequences of this theory.



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## I. INTRODUCTION

For thousands of years great minds of mankind tried to find the most fundamental element in nature such as four elements, atoms, quarks, and strings. Nowadays, there is a hope that this quest will eventually lead us to a 'theory of everything' reconciling general relativity with quantum mechanics and other forces. In this perspective, configurations of a fundamental object such as a superstring should represent all known particles and their species. However, to have configurations, the object should have some internal structure and this implies that the object should consist of even smaller objects. This brings us an obvious logical paradox.

On the other hand, there is a long history of the conception that the universe is actually made of abstract entities like logic rather than material objects. A famous example is the Pythagorean who believed that numbers are fundamental constituents of the nature. Interestingly, recent developments of quantum information science revealed that abstract information can play a fundamental roll in the physical world. This idea can be represented by an implicative slogan in quantum information community, "It from Bit!"

There are many observations supporting the slogan. For instance, it was shown that quantum mechanics and special relativity miraculously cooperate so as not to allow super-luminal information transfer (See for example [1]), and this no-signalling condition could be a basic principle of physics. Furthermore, Landauer's principle [2] stating that erasing information requires energy consumption implies an intrinsic relation between information and energy. It was also suggested that wavefunctions in quantum mechanics actually represent information of a system [3] or relations [4] rather than particles or waves.

Studies of black hole physics after Bekenstein and Hawking have consistently implied that there is a deep connection among gravity, thermodynamics and information [5]. Recently proposed Verlinde's idea [6] linking gravity to entropic force enhance this viewpoint, because entropy can be interpreted as a measure of information. He derived Newton's second law and Einstein's equation from the relation between the number of degrees of freedom $N$ of a holographic screen and energy $E \sim NT$ in a volume enclosed by the screen. Here, $T$ is the temperature of the screen. Padmanabhan [7] also proposed that classical gravity can be derived from the equipartition energy of horizons. These works, influenced by Jacobson's proposal that Einstein equation describes the first law of thermodynamics at local Rindler horizons, attracted much interest in community [8–22]. All these works emphasize mainly the connection between thermodynamics and gravity.

On the other hand, in a series of works [23], the author and co-workers (LLK hereafter) suggested that information plays a crucial role in gravitational systems such as dark energy and black holes. For example, in 2007 LLK presented a new idea that a cosmic causal horizon with a radius $r \sim O(1/H)$ could have Hawking temperature $T_h \propto 1/r$, quantum information theoretic entropy $S_h \propto r^2$ represented by bits, and hence, a kind of thermal energy $E_h \sim T_h S_h \propto r$, which can be identified to be the dark energy with density $\rho_h \sim r^{-2} \sim O(M_P^2 H^2)$. Here, $M_P$ is the Planck mass and $H$ is the Hubble parameter. We set the Boltzman constant $k_B = 1$. This energy corresponds to the quantum vacuum energy of a spatial region bounded by the horizon and is related to information erasing process due to the expanding cosmic horizon and also possibly to quantum entanglement of the vacuum. LLK also suggested that a black hole mass has a similar quantum information theoretic origin, and that Jacobson's formalism about Einstein gravity actually represents information loss process at local Rindler horizons in a curved spacetime.

Since entropy is usually proportional to $N$, there is a clear similarity between this informational energy $E_h$ and the equipartition energy $E$ considered by Verlinde. However, there are also some differences between two approaches which will be shown below.

In this paper, based on these works, it is suggested that major physics such as quantum mechanics, Einstein gravity and Newton's mechanics are simply describing information processing at causal horizons.

## II. IT FROM BIT

Let me start by summarizing some well-known physical principles and laws.

1. Landauer's principle: To erase information $dS$, at least energy $dE = TdS$ should be consumed.
   $\rightarrow$ Information is related to thermal Energy

2. $E = mc^2$: Energy is related to Mass (matter)

3. Einstein Equation, $G_{\mu\nu} = 8\pi G\, T_{\mu\nu} \rightarrow$ Matter generates Gravity

4. Unruh effect: Quantum fluctuation looks thermal to some observers

Now, in a very naive language, my observations can be summarized as follows. By combining the principles 1 and 2, one can see "Matter is related to information". On the other hand, 1+2+3 implies "Gravity is related to information".

1+4 means "Quantum mechanics is related to information". Although these propositions should be justified by more rigorous reasonings, this brief argument shows the essence of the idea.

Inspired by the above principles and laws, it is now suggested that we can choose the followings as new and general guiding principles which any physical law is based on.

• Guiding principles

1. *General equivalence: All systems of reference (coordinates) are equivalent for formulating physical laws regardless of their motions.*

2. *Information has a finite density and speed; the quantity of information contained in a finite object is finite, and there is a maximal speed of classical information propagation, namely, the light velocity.*

3. *Information is fundamental: Physical laws regarding an object (matter or spacetime) should be such that they respect observers' information about the object.*

Note first that we are assuming neither quantum mechanics nor Einstein gravity. They shall be derived from above assumptions. We need to assume the metric nature of spacetime and ignore any fluctuation of spacetime.

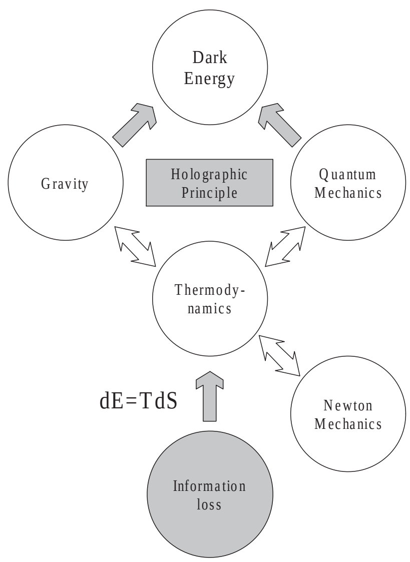


FIG. 1. Relation between various physics fields. Information seems to be the root of all physics. Causal horizons for some observers act as an information barrier, and thermodynamics occurs as a result. Then, an information-energy (i.e., entropy-energy) relation $dE = TdS$ at the horizons lead to main equations of physics.

Some of these assumptions deserve more explanation. Finite information propagation velocity implies that there is an information barrier in spacetime for some observers. This barrier could be, for example, a Rindler horizon, a black hole event horizon or a light cone. Thus, there could be a situation where matter (particles or waves) crosses the causal horizon for an observer. Then, the observer can get no more information about the phase space (position and velocity) of the matter. It is reasonable that this ignorance of the observer about the matter should be represented by the increase of the information theoretic entropy $S$ (for example Shannon entropy or entanglement entropy) of the horizon. According to the holographic principle this should be accompanied by the horizon area increase. Furthermore, due to Landauer's principle or the second law of thermodynamics, there should be some kind of 'thermal energy' $dE = TdS$. That means the usual first law of thermodynamics in gravitational systems is actually the second law disguised. Major physical laws such as Einstein equation and Newton's equation seem to simply represent this information-energy relation.

In short I suggest the following conjecture.

• Conjecture: *Main physical laws simply describe the phase space information loss of matter or space time crossing a causal horizon for an observer*

From this conjecture, the following results are derived (See Fig. 1.). Quantum mechanics arises from ignorance of observer outside of a causal horizon about matter inside the horizon (section III). For an accelerating Rindler observer relative to a particle this leads to Newton's second law as in Verlinde's formalism (section IV). For a local inertial frame in curved spacetime the conjecture leads to Einstein equation through Jacobson's formalism and naturally explain the origin of Verlinde's entropic gravity (section V). This theory also explains the origin of holography and quantum entanglement (section VI). Finally, if we apply the conjecture to a cosmic causal horizon, we obtain dark energy comparable to observed one and zero cosmological constant (section VII).

## III. QUANTUM MECHANICS FROM INFORMATION LOSS

In this section it will be shown that quantum field theory (QFT), and hence quantum physics, is not fundamental and can be derived by considering phase space information loss of matter crossing Rindler horizons [24]. Let us begin by considering an accelerating Rindler observer $\Theta_R$ with acceleration $a$ in $x_1$ direction in a flat spacetime with coordinates $X = (t, x_1, x_2, x_3)$ (See Fig. 1). The Rindler coordinates $\xi = (\eta, r, x_2, x_3)$ for the observer are defined with

$$ct = r\ sinh(a\eta/c),\ x_1 = r\ cosh(a\eta/c) \tag{1}$$

on the Rindler wedges. There is an inertial Minkowski observer $\Theta_M$ too. Now, consider a field $\phi$ flowing across the Rindler horizon at a point $P$ and entering the future wedge $F$. A configuration for $\phi(x, t)$ is not necessarily meant to be classical but to be just some physical function of spacetime. It is important to note that in this theory the field $\phi$ cannot have a specific value before measurements according to our assumptions, unless the relevant observer gets the information about the field value in advance.

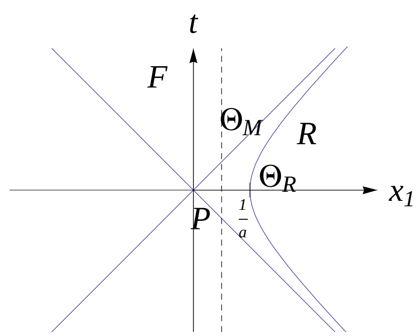


FIG. 2. Rindler chart for the observer $\Theta_R$ (curved line), who has no accessible information about field $\phi$ in a causally disconnected region $F$. Thus, the observer can only estimate a probabilistic distribution of the field, which turns out to be thermal and equal to that of a quantum field for inertial observer $\Theta_M$ (dashed line) in Minkowski spacetime.

As the field enters the Rinder horizon for the observer $\Theta_R$, the observer shall not get information about phase space information (configurations and momentums) of $\phi$ any more and all what the observer can expect about $\phi$ evolution beyond the horizon is a probabilistic distribution $P[\phi]$ of $\phi$ beyond the horizon. Already known information about $\phi$ acts as constraints for the distribution. I suggested that this ignorance is the origin of quantum randomness. Physics in the F wedge should reflect the ignorance of the observer in the R wedge, if information is fundamental [24].

One constraint comes from the energy conservation

$$\sum_{i=1}^{n} P[\phi_i]H(\phi_i) = E, \tag{2}$$

where $H(\phi_i)$ is the Hamiltonian as a function of the i-th configuration of the field $\phi_i$ and $E$ is its expectation. Another one is the unity of the probabilities $\sum_{i=1}^{n} P[\phi_i] = 1$. Then, using Boltzmann's principle of maximum entropy one can calculate the probability distribution estimated by the Rindler observer

$$P[\phi_i] = \frac{1}{Z} \exp\left[-\beta H(\phi_i)\right], \tag{3}$$

where $\beta$ is the Lagrangian multiplier, and the partition function is

$$Z = \sum_{i=1}^{n} \exp\left[-\beta H(\phi_i)\right] = tr\ e^{-\beta H}, \tag{4}$$

where the trace is assumed to be performed with a (classical) discrete vector basis. Lisi showed a related derivation of the partition function by assuming a universal action reservoir for information [25].

From now on, let us consider a continuum limit for a scalar field $\phi$ with Hamiltonian

$$H(\phi) = \int d^3x \left[ \frac{1}{2} \left( \frac{\partial \phi}{\partial t} \right)^2 + \frac{1}{2} (\nabla \phi)^2 + V(\phi) \right] \tag{5}$$

and potential $V$. For the Rindler observer with the coordinates $(\eta, r, x_2, x_3)$ the proper time variance is $ard\eta$ and hence the Hamiltonian is changed to

$$H_R = \int_{r>0} drdx_\perp \ ar[\frac{1}{2} \left( \frac{\partial \phi}{ar\partial \eta} \right)^2 + \frac{1}{2} \left( \frac{\partial \phi}{\partial r} \right)^2 + \frac{1}{2} (\nabla_\perp \phi)^2 + V(\phi)], \tag{6}$$

where $\perp$ denotes the plane orthogonal to $(\eta, r)$ plane. Then, Eq. (4) becomes Eq. (2.5) of Ref. [26];

$$Z_R = tr \ e^{-\beta H_R}. \tag{7}$$

It is important to notice that $Z$ (and hence $Z_R$) here is not a quantum partition function but a statistical partition function corresponding to the uncertain field configurations beyond the horizon.

Unruh showed [26] that the real-time thermal Green's functions of the Rindler observer with $Z_R$ are equivalent to the vacuum Green's function in Minkowski coordinates. Thus, as well known, the Minkowski vacuum is equivalent to thermal states for the Rindler observers. What is newly shown in Ref. [24] is that the thermal partition function $Z_R$ assumed in Ref. [28] is actually from the phase space information loss beyond the Rindler horizon and, therefore, the QFT formalism is equivalent to the purely information theoretic formalism. Recall that Eq. (7) was derived without using any quantum physics. Since quantum mechanics can be thought to be a single particle approximation of QFT, this implies also that particle quantum mechanics emerges from information theory applied to Rindler horizons and is not fundamental. Another important point here is that thermal nature of quantum field is due to the information loss and can be treated as more fundamental than the quantum nature.

This theory explains why superluminal communication is impossible even using quantum nonlocality (entanglement). Quantum randomness and hence quantum correlation originates from the very fact that information cannot be sent faster than light.

In Ref. [28] it was shown by analytical continuation that in the Rindler coordinates $Z_R$ is *mathematically* equivalent to

$$Z_R = N_0 \int_{\phi(0)=\phi(\beta')} D\phi \ exp\{-\alpha \int_0^{\beta'} d\tilde{\eta} \int_{r>0} drdx_\perp \ ar \left[ \frac{1}{2} \left( \frac{\partial \phi}{ar\partial \tilde{\eta}} \right)^2 + \frac{1}{2} \left( \frac{\partial \phi}{\partial r} \right)^2 + \frac{1}{2} (\nabla_\perp \phi)^2 + V(\phi) \right] \}, \tag{8}$$

where we introduced a constant $\alpha$ having a dimension of $1/H_R t$ and $\beta \equiv \alpha\beta'$.

By further changing integration variables as $\tilde{r} = rcos(a\tilde{\eta}), \tilde{t} = rsin(a\tilde{\eta})$ and choosing $\beta' = 2\pi/a \equiv 1/\alpha T_U$ the region of integration is transformed from $0 \leq \tilde{\eta} \leq \beta', 0 \leq r \leq \infty$ into the full two dimensional $\tilde{t} - \tilde{r}$ space. This $\beta'$ value leads to Unruh temperature $T_U = a/2\alpha\pi$. From the well-known QFT result, one can find $1/\alpha = \hbar$. This means that the Planck constant $\hbar$ is some fundamental temperature given by nature.

Then, the partition function becomes

$$Z_Q^E = N_1 \int D\phi \ exp \left\{ -\frac{I_E}{\hbar} \right\}. \tag{9}$$

where $I_E$ is the Euclidean action for the scalar field in the inertial frame. By analytic continuation $\tilde{t} \to it$, one can see $Z_Q^E$ becomes the usual zero temperature quantum mechanical partition function $Z_Q$ for $\phi$. Since both of $Z_R$ and $Z_Q$ can be obtained from $Z_Q^E$ by analytic continuation, they are physically equivalent as pointed out in Ref. [28].

It is straightforward to extend the previous analysis to quantum mechanics for point particles. We can imagine a point particle at a point $P$ just crossing the Rindler horizon and entering the future wedge $F$. The maximal ignorance of the observer about the particle is represented by probability distribution $P[x_i(t)]$ for the i-th possible path that the particle may take. Then, the partition function is

$$Z_R = \sum_{i=1}^{n} \exp\left[-\beta H(x_i)\right] = tr \ e^{-\beta H}, \tag{10}$$

where $H$ is the point particle Hamiltonian now. Since the usual point particle quantum mechanics is a non-relativistic and single particle limit of the quantum field theory, we expect $Z_R$ is equal to the quantum partition function for the particle with mass $m$ in Minkowski spacetime

$$\begin{aligned} Z_Q &= N_2 \int Dx \exp\left[-\frac{i}{\hbar}\int d\tilde{t}\left\{\frac{m}{2}\left(\frac{\partial x}{\partial \tilde{t}}\right)^2 - V(x)\right\}\right] \\ &= N_1 \int Dx\ exp\left\{-\frac{i}{\hbar} I(x_i)\right\}, \end{aligned} \tag{11}$$

where $I$ is the action for the point particle. Then, as is well known one can associate each path $x_i$ with a wave function $\psi \sim e^{-iI}$, which leads to Schrödinger equation for $\psi$ [29]. Therefore, our theory explains naturally the origin of path integral and the similarity between quantum mechanical formalism and statistical physics.

## IV. NEWTON MECHANICS FROM INFORMATION LOSS

Quantum mechanics of the previous section, of course, leads to classical Newton mechanics for an appropriate limit ($\hbar \to \infty$ and $c \to \infty$ ). Alternatively, one can also directly derive Newton mechanics from the information-energy relation based on the partition function as in Verlinde's approach. If our theory is sound, two approaches should give a same description.

The free energy $G$ from the partition function of the previous section can be expressed as

$$G = -\frac{1}{\beta} ln Z_R. \tag{12}$$

The classical path $x_{cl}$ for the particle corresponds to the saddle point ($Z_R \sim exp[-\beta I_E(x_{cl})]$ ) [30], where $I_E(x_{cl})$ is the Euclidean action for classical path satisfying the Lagrange equation. In this limit the free energy becomes

$$G \simeq G_{cl} = -\frac{1}{\beta}\left(-\beta I_E(x_{cl})\right) + C = I_E(x_{cl}) + C, \tag{13}$$

where $C$ is a constant. Since the maximum entropy is achieved when $G$ is minimized, we see that classical physics with the minimum action corresponds to a maximum entropy condition. In other words, the classical path is the typical path maximizing the Shannon entropy $h[P]$ regarding the phase space information with the constraints for the Rindler observer.

Therefore, one can find that the entropy associated with the thermodynamical interpretation of mechanics and gravity is related to information of matter crossing the horizon. For fixed temperature, pressure and volume, the minimum free energy condition $dG = 0$ is equivalent to $dE - TdS = 0$, i.e., the first law of thermodynamics or the information-energy relation, $dE = TdS$. This explains why classical physics can be obtained from thermodynamics as in Verlinde's approach. The maximum entropy proposal in Verlinde's theory can be easily explained in this theory.

To be concrete, consider an accelerating test point particle with acceleration $a$ and mass $m$ (Fig. 1) and an observer $\Theta_R$ at rest at the instantaneous distance $\Delta x$ from the particle. If we accept the general principle of relativity stating that all systems of reference are equivalent regardless of their motions, we can imagine an equivalent situation where the particle is at rest and the observer $\Theta_R$ accelerates in the opposite direction with acceleration $-a$.

The key idea is that for an accelerating object there is always such an observer that the object seems to cross a Rindler horizon of the observer. For this observer there is the phase space information loss, and hence, some thermal energy associated. If the observer is at a specific distance $\Delta x = c^2/a$, the observer could see the particle just crossing his or her Rindler horizon. Then, the Rindler horizon hides the information of the particle and this leads to information loss, which should be compensated by an increase of the entropy $S_h$ of the Rindler horizon. This distance $\Delta x = c^2/a$ is special, because, for the observer there, $\tau$ becomes a proper time and the Rindler Hamiltonian becomes a physical one generating $\tau$ translation. Since the horizon is a Rindler horizon, we can safely use the Unruh temperature

$$T_U = \frac{\hbar a}{2\pi c} \tag{14}$$

for the horizon.

Then, if we accept the holographic principle, it is natural to think that the mass of the test particle $m$ is converted to the horizon energy $\Delta E_h$. In our theory $E_h$ is simply the total energy inside the horizon. Therefore, the following relations

$$mc^2 = \Delta E_h = T_U \Delta S_h = \frac{\hbar a}{2\pi c}\Delta S_h \tag{15}$$

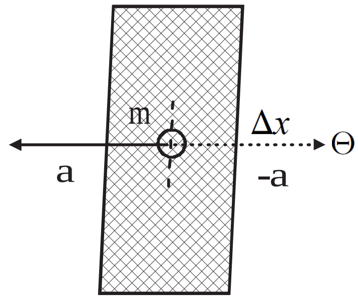


FIG. 3. A test particle with mass $m$ is accelerating with acceleration $a$ with respect to an observer instantaneously resting at $\Delta x$ from the particle. Alternatively we can imagine that the particle is at rest and the observer moves in the opposite direction with acceleration $-a$. The observer could see the particle crossing a Rindler horizon (the shaded plane).

should hold, which implies for $a = c^2/\Delta x$

$$\Delta S_h = \frac{2\pi cm\Delta x}{\hbar}, \tag{16}$$

i.e, Eq. (3.6) of Verlinde's paper. Similarly, Culetu [18] pointed out the role of the specific distance $\Delta x$ in Verlinde's formatlism.

There have been criticisms [18, 33–35] on this entropy variation formula in Verlinde's original model. The difficulty disappears in our theory, where we identify the Rindler horizon as Verlinde's holographic screen and the entropy of the Rindler horizon $S_h$ as the entropy of the screen $S$.

Then, one can define the holographic entropic force

$$F = \frac{\Delta E_h}{\Delta x} = T_U \frac{\Delta S_h}{\Delta x} = ma, \tag{17}$$

which is just Newton's second law.

In short, from the viewpoint of our theory, Verlinde's holographic screen corresponds to Rindler horizons and its entropy is associated with the lost phase space information of the particle crossing Rindler horizons [36]. Then, there is an entropic force linked to this information loss which can be calculated. Thus, our theory reproduces and supports Verlinde's mathematical formalism basically. However, there are several differences between Verlinde's model and our theory, which will be shown in the next section. Interestingly, this new interpretation seems to also give a hint for the origin of inertia and mass. The inertia of the particle can be interpreted as resistance from the horizon dragging which the external force feels. This dragging force is proportional to acceleration, hence, $F = ma$.

We see that inertia and Newton's second law have something to do with Rindler horizons and information loss at the horizons. In our formalism and Verlinde's formalism, inertial mass and gravitational mass have a common origin and hence equivalent. This is consistent with the Einstein's equivalence principle.

## V. EINSTEIN GRAVITY FROM INFORMATION LOSS

Similarly, one can interpret Jacobson's formalism and Verlinde's entropic gravity in terms of information at Rindler horizons [36]. The equivalence principle allows us to choose an approximately flat patch for each spacetime point. According to the principle one can not locally distinguish the free falling frame from a rest frame without gravity. Therefore, we can again imagine an accelerating observer $\Theta$ with acceleration $-a$ respect to the test particle in the rest frame of the particle. If $\Delta x = c^2/a$, the test particle is just at the Rindler horizon for the observer $\Theta$, and there should be energy related to entropy change, i.e., $dE_h = TdS_h$.

Following Jacobson we can generalize this information-energy relation by defining the energy flow across the horizon $\Sigma$

$$dE = -\kappa\lambda \int_\Sigma T_{\alpha\beta}\xi^\alpha d\Sigma^\beta \tag{18}$$

where $d\Sigma^\beta = \xi^\beta d\lambda dA$, $dA$ is the spatial area element, and $T_{\alpha\beta}$ is the energy momentum tensor of matter distribution. Using the Raychaudhuri equation one can denote the horizon area expansion $\delta A \propto dS_h$ and the increase of the entropy

as

$$dS_h = \eta\delta A = -\eta\lambda \int_\Sigma R_{\alpha\beta}\xi^\alpha d\Sigma^\beta, \tag{19}$$

with some constant $\eta$ [37]. If $S_h$ saturates the Bekenstein bound, $\eta = c^3/4\hbar G$.

Inserting Eqs. (18) and (19) into $dE = T_U dS_h = \hbar\kappa dS_h/2\pi c$ one can see $2\pi c T_{\alpha\beta}\xi^\alpha d\Sigma^\beta = \eta R_{\alpha\beta}\xi^\alpha d\Sigma^\beta$. For all local Rindler horizons this equation should hold. Then, this condition and Bianchi indentity lead to the Einstein equation

$$R_{\alpha\beta} - \frac{Rg_{\alpha\beta}}{2} + \Lambda g_{\alpha\beta} = \frac{2\pi}{\eta c} T_{\alpha\beta} \tag{20}$$

with the cosmological constant $\Lambda$ as shown in Jacobson's paper.

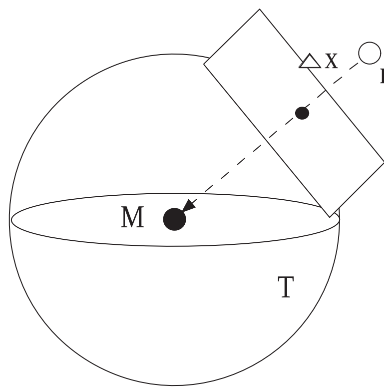


FIG. 4. A test particle with mass $m$ is free falling with acceleration $a$ at distance $r$ from a massive object with mass $M$ at the center. Consider an equivalent situation where is an accelerating observer $\Theta$ with acceleration $-a$. If the observer is instantaneously at the distance $\Delta x = c^2/a$ from the test particle, the observer could see the particle crossing the local Rindler horizon (the dashed line) for the observer.

Of course, one can derive Newton's gravity from the above Einstein equation. Alternatively, it is also meaningful to derive Newton's gravity from $dE = TdS$ relation and to show that our approach fills the gap between Jacobson's formalism and Verlinde's entropic gravity.

Consider an observer instantaneously at the distance $\Delta x$ from the test particle with mass $m$. We can consider a set of such observers surrounding the central mass $M$ at the distance $r + \Delta x$ from the center. I suggested that the holographic screen considered by Verlinde can be interpreted to be an imaginary overlap of these local Rindler horizons with a same Unruh temperature $T_U$ for the observers (Fig. 4). Again, the mass of the test particle $m$ should be converted to the horizon energy $E_h$ and this induces the increase of the horizon entropy $\Delta S_h$ eventually. Therefore, one can see relations

$$mc^2 = \Delta E_h = T_U \Delta S_h = \frac{\hbar a}{2\pi c}\Delta S_h. \tag{21}$$

Using the relation $\Delta x = c^2/a$ above, one can obtain the entropy change in Eq. (16) again. Inspired by the holographic principle we assume that mass inside a region is equal to the horizon energy, that is,

$$Mc^2 = E_h = 2T_h S_h = 2T_U S_{BH}, \tag{22}$$

where the horizon energy relation $E_h = 2T_h S_h$ [38] and the Bekenstein bound (i.e., $S_h = S_{BH}$) were used. The Bekenstein-Hawking entropy

$$S_{BH} = \frac{c^3\, A}{4G\hbar} \tag{23}$$

is a bound of information in a region of space with a surface area $A$ [5]. Since it was shown that the entropy of a Rindler horizon is equal to one quarter the area of the horizon in Planck units, this choice is reasonable. From this equation one can obtain $T_U = Mc^2/2S_{BH}$ and the acceleration

$$a = \frac{2\pi c T_U}{\hbar} = \frac{GM}{r^2}. \tag{24}$$

Then, from Eq. (16) and the above equation, the entropic force is given by

$$F = T_U \frac{\Delta S}{\Delta x} = \frac{GMm}{r^2}, \tag{25}$$

which is just Newton's gravity.

Therefore, we conclude that the holographic screens at a given position in Verlinde's formalism are actually Rindler horizons at the position for specific observers accelerating relative to the test particle. This identification could easily explain many questions on Verlinde's formalism and provide better grounds for the theory. The entropy-distance relation holds only for specific observers, and the use of Unruh temperature is valid, because the holographic screen can be actually a set of Rindler horizons. This shows the interesting connection between Jacobson's model [37] or the quantum information theoretic model [32, 39] to Verlinde's model.

However, there are also several distinctions between Verlinde's entropic gravity and our information theoretic model. First, in Verlinde's work, the screen bounds the emerged part of space, and the approaching particle eventually merges with the microscopic degrees of freedom on the screen. In our theory, spacetime is not necessarily emergent and the particle just crosses the horizon and entropy is related to the phase space information loss. Second, in his theory, the entropy of the screen changes as the particle approaches to the screen, and the screen should move appropriately to satisfy the entropy formula, while in our theory the change is due to information loss at Rindler horizons of specific observers. Third, in Verlinde's theory the holographic screens correspond to *equipotential* surfaces, while in our theory they correspond to *isothermal* Rindler horizons (i.e., with the same $|a|$). Finally, since Rindler horizons are observer dependent, there is no objective or observer-independent notion of the Rindler horizon entropy increase in our theory. This help us to avoid the issue of the time reversal symmetry breaking in entropic gravity. These differences help us to resolve the possible difficulties of Verlinde's original model [40] and to understand the connection between gravity and information. Compared to other models, our theory emphasizes the role of information rather than thermodynamics.

## VI. HOLOGRAPHY AND ENTANGLEMENT FROM INFORMATION LOSS

In this section, I explain how the holographic principle and quantum entanglement can arise from the information loss. The information theoretic derivation of quantum mechanics in the section III makes it simple to understand the information theoretic origin of the holographic principle [43]. Consider a $d+1$-dimensional bulk region $\Omega$ with a $d$-dimensional boundary $\partial\Omega$ that is an one-way causal horizon (see Fig. 5). An outside observer $\Theta_O$ can not access the information about matter or spacetime in the region due to the horizon. The best the observer could do is to estimate the probability of each possible field configuration of $\phi$ in $\Omega$, which turns out to be the probability amplitude in the path integral. During this estimation, $\Theta_O$ would use the maximal information available to her/him.

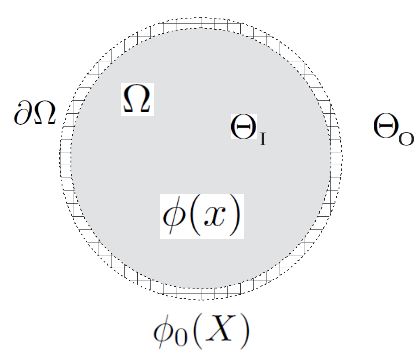


FIG. 5. A bulk $\Omega$ with a causal horizon $\partial\Omega$ and an inside observer $\Theta_I$. The outside observer $\Theta_O$ has no information about the phase space of $\phi(x)$ in $\Omega$ except for its boundary values $\phi_0(X)$ and derivatives. Thus, according to our information theoretic interpretation, the physics in $\Omega$ is completely described by the boundary physics on $\partial\Omega$, which is just the holographic principle.

According to the postulates, there is no non-local interaction that might allow super-luminal communication. Therefore, we restrict ourselves to local field theory. For a local field, any influence on $\Omega$ from the outside of the horizon should pass the horizon. This means that all the physics in the bulk $\Omega$ is fully described by the degrees of the freedom (DOF) on the boundary $\partial\Omega$, which is just the essence of the holographic principle! Information loss due to a horizon allows the outside observer $\Theta_O$ to describe the physics in the bulk using only the DOF on the boundary. That is the best $\Theta_O$ can do by any means, and the general equivalence principle demands that this description is sufficient for understanding the physics in the bulk, which is equivalent to the holographic principle.

Therefore, the following version of the holographic principle is a natural consequence of the information theoretic formalism of QFT based on our postulates.

**Theorem 1 (holographic principle)** *For local field theory, physics inside an 1-way causal horizon can be described completely by physics on the horizon.*

Note that this derivation is generic, because the arguments we used in this section rely on neither the specific form of the metric nor any symmetries the fields may have.

Now, how can entanglement arise in this theory [44]? Let us assume that the bulk has $N_B$ bits while the surface has $N_b$ bits. According to the holographic principle, the bulk has only area-proportional DOF and hence there should be redundancy in the bulk bits $B_\alpha$. Therefore, they are not independent of each other. We can not simply ignore some of the bulk bits, because the boundary bits should be able to reproduce arbitrary configuration of the bulk bits, at least probabilistically. Therefore, only possible way seems to be $n$ to 1 correspondence between the bulk bits and the boundary bits. Mathematically, this could mean that there is a $2^{N_B}$ to $2^{N_b}$ mapping $f : 2^{N_B} \to 2^{N_b}$. Since the boundary bits should fully describe the bulk bits (at least probabilistically), this mapping should be a surjective function.

As a toy example, consider a combination of two bulk bits $B_1$ and $B_2$ which is described by a single common boundary bit $b_0$ such that both of $(B_1, B_2) = (0, 0)$ and $(B_1, B_2) = (1, 1)$ correspond to $b_0 = 0$ and both of $(B_1, B_2) = (1, 0)$ and $(B_1, B_2) = (0, 1)$ correspond to $b_0 = 1$. Some information in the bulk bits is lost during the mapping. (This reminds us of the information loss process considered by 't Hooft in the quantum determinism proposal [45]. He introduced equivalence classes of states that evolve into one and the same state.) Now, assume that $b_0 = 1$. This specific mapping can be represented by a matrix relation

$$\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}_B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}_b, \tag{26}$$

where the vector on the left represents the bulk bits in the basis $(00, 01, 10, 11)$ and the vector on the right represents the boundary bits in the basis $(0, 1)$, respectively. The 4 by 2 matrix represents $f$. With only $b_0$ value the outside observer can not distinguish two cases $(B_1, B_2) = (1, 0)$ and $(B_1, B_2) = (0, 1)$. Thus, the statistical probability of $b_0$ estimated by the outside observer should be an addition of two probabilities,

$$P_b = P_B((1, 0)) + P_B((0, 1)), \tag{27}$$

where $P_B((1, 0)) = 1/2 = P_B((0, 1))$ is the probability that $(B_1, B_2) = (1, 0)$ and $P_b = 1$ is the probability that $b_0 = 1$. In the path integral formalism derived previously, for the inside observer this probability corresponds to an entangled quantum state

$$\psi = \frac{1}{\sqrt{2}}(|1\rangle|0\rangle + |0\rangle|1\rangle). \tag{28}$$

Therefore, quantum entanglement is a natural consequence of the holographic principle. In the information theoretic formalism described in the section III, a quantum state in the bulk corresponds to a statistical probability like $P_B$ estimated by the outside observer who sees the causal horizon. This formalism could explain the correspondence between $P_b$ and $\psi$.

## VII. DARK ENERGY FROM INFORMATION LOSS

Before Verlinde's proposal LLK suggested an idea that dark energy is related to information content of the cosmic horizon [23, 31, 32, 41, 42]. If the cosmic causal horizon has a radius $r \sim O(H^{-1})$, Hawking temperature $T \sim 1/r$, and entropy $S \sim r^2$, there could be a kind of thermal energy $E \sim TS \sim r$ corresponding to the vacuum energy, dubbed 'quantum informational dark energy' by the authors. Here $H = da/adt$ is the Hubble parameter with the scale factor $a$ and the cosmic horizon could be the event horizon, the Hubble horizon or the apparent horizon. (There appeared similar dark energy models based on the Verlinde's entropic gravity [33, 46–48].)

To calculate the horizon energy $E_h$ as vacuum energy of the universe, let us consider a generic holographic entropy for a causal cosmic horizon with radius $r$,

$$S = \frac{\eta c^3 r^2}{G\hbar}, \tag{29}$$

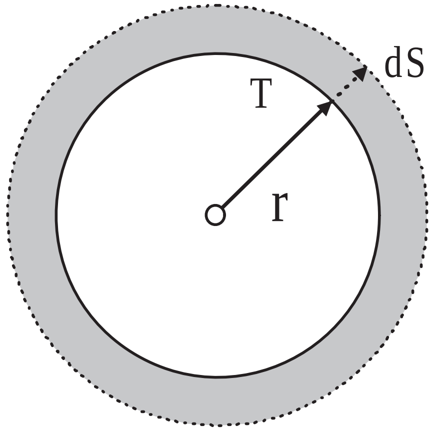


FIG. 6. Expansion of a cosmic horizon $\Sigma$ with a radius $r$ and the Hawking temperature $T$ induces the information erasing of the gray region with entropy $dS$. This information erasing consumes the energy $TdS$, which can turn into dark energy finally.

and

$$T = \frac{\epsilon \hbar c}{r}, \tag{30}$$

with parameters $\eta$ and $\epsilon$. For the Hawking-Gibbons temperature $\epsilon = 1/2\pi$, and for the Bekenstein entropy $\eta = \pi$.

Now, one can calculate the vacuum energy using the holographic principle. By integrating $dE$ on the isothermal surface $\Sigma$ of the causal horizon with Eqs. (29) and (30), we obtain the horizon energy

$$E = \int_\Sigma dE = T \int_\Sigma dS = \frac{\eta \epsilon c^4 r}{G}. \tag{31}$$

Another possible interpretation is that this is the energy of the cosmic Hawking radiation [49]. Then, the energy density due to $E_h$ is given by

$$\rho_h = \frac{3E}{4\pi r^3} = \frac{6\eta\epsilon c^3 M_P^2}{\hbar r^2} \equiv \frac{3d^2 c^3 M_P^2}{\hbar r^2}, \tag{32}$$

which has the form of the holographic dark energy [50]. This kind of dark energy was also derived in terms of entanglement energy [23] and quantum entanglement force [39]. From the above equation we immediately obtain a formula for the constant

$$d = \sqrt{2\eta\epsilon}, \tag{33}$$

which is the key parameter determining the characteristic of holographic dark energy. The simplest choice is such that $S_h$ saturates the Bekenstein bound and $T_h$ is the Hawking-Gibbons temperature $\hbar c/2\pi r$. Then, $\eta\epsilon = 1/2$ and $d = 1$, which is favored by observations and theories [51, 52]. Thus, the holographic principle applied to a cosmic causal horizon naturally leads to the holographic dark energy with $d = 1$ [39]!

From the cosmological energy-momentum conservation equation, one can obtain an effective dark energy pressure [50]

$$p_{DE} = \frac{d(a^3 \rho_h(r))}{-3a^2 da}, \tag{34}$$

from which one can derive the equation of state. To compare predictions of our theory with current observational data, we need to choose the horizon. The event horizon is the simplest one, if there is no interaction term between dark energy and matter [50]. In this case one can find the equation of state for holographic dark energy as a function of the redshift $z$ [50];

$$\begin{aligned} \omega_{DE} &= \left(1 + \frac{2\sqrt{\Omega_\Lambda^0}}{d}\right)\left(-\frac{1}{3} + z\frac{\sqrt{\Omega_\Lambda^0}(1-\Omega_\Lambda^0)}{6d}\right) \\ &\simeq w_0 + w_1(1-a), \end{aligned} \tag{35}$$

where the current dark energy density parameter $\Omega^0_\Lambda \simeq 0.73$ [50, 53]. For $d = 1$ these equations give $w_0 = -0.903$ and $w_1 = 0.208$. According to WMAP 7-year data with the baryon acoustic oscillation, SN Ia, and the Hubble constant yields $w_0 = -0.93 \pm 0.13$ and $w_1 = -0.41^{+0.72}_{-0.71}$ [54]. Thus, the predictions of our theory well agree with the recent observational data. If we use an entanglement entropy calculated in [39] for $S_h$, one can obtain $d$ slightly different from 1.

It was also shown that holographic dark energy models with an inflation with a number of e-folds $N_e \simeq 65$ can solve the cosmic coincidence problem [50, 55] thanks to a rapid expansion of the event horizon during the inflation.

Following [39] and [48] one can obtain an entropic force for the dark energy

$$F_h \equiv \frac{dE_h}{dr} = \frac{c^4 \eta \epsilon}{G}, \tag{36}$$

which could be also identified as a 'quantum entanglement force' dubbed by LLK, if $S_h$ is the entanglement entropy.

It is simple to see why the cosmological constant $\Lambda_c$ should be zero. The classical cosmological constant $\Lambda_c$ appears in the gravity action as

$$S = \int d^4x \sqrt{-g}(R - 2\Lambda_c). \tag{37}$$

It is usually argued that after taking vacuum expectation of quantum fields, the Friedmann equation has additional contribution $\Lambda_q = \rho_q / M_P^2 c^2$ from the vacuum quantum fluctuation $\rho_q$. Thus, the total cosmological constant becomes $\Lambda = \Lambda_c + \Lambda_q$, and the total vacuum energy density is given by

$$\rho_{vac} = M_P^2 c^2 (\Lambda_c + \Lambda_q). \tag{38}$$

Without a fine tuning it is almost impossible for two terms to cancel each other to reproduce the tiny observed value, which is the well-known cosmological constant problem.

A constant $\Lambda_c$ results in vacuum energy proportional to $\Lambda r^3$ clearly violating the holographic principle for large $r$ (where matter energy density of the universe is small), because $E_h \propto r$ according to the principle and the information-energy relation. This implies that the 'time independent' classical cosmological constant $\Lambda_c$ should be zero and $\Lambda_q$ is proportional to $\rho_h$ in Eq. (32), unless there is interaction between matter and dark energy. Of course, this argument does not show how to remove the cosmological constant explicitly in QFT. QFT is not one of our assumptions but derived with specific conditions. Since the holographic principle is in contradiction with QFT at a large scale, this might mean that we need to change QFT at a cosmological scale.

In summary, in this theory the dark energy density is small due to the holographic principle, comparable to the critical density due to the $O(1/H)$ horizon size or $N_e \simeq 65$, and non-zero due to quantum vacuum fluctuation. The holographic principle also demands that the cosmological constant is zero.

## VIII. DISCUSSION

In short, the Einstein equation links matter to gravity and his famous formula $E = mc^2$ links matter to energy. We know also that the Landauer's principle links information to energy. Thus, now we have relations among information, gravity, quantum mechanics and classical mechanics. Our theory implies that physical laws are more about information rather than particles or waves. Quantum randomness and its thermal nature arise from information loss at causal horizons. This gives us a new hint of quantum gravity. Our new approach also shows interesting connections between Jacobson's model [37], the quantum information theoretic model [32, 39] and Verlinde's model for gravity.

We also see that inertia and Newton's second law have something to do with Rindler horizons and information loss at the horizons. In our formalism and Verlinde's formalism, inertial mass and gravitational mass have a common origin and hence equivalent.

The holographic principle and quantum entanglement can be explained easily in this formalism. All these studies are not a simple reinterpretation of existing physics. If information really is the essence of the universe, this alters our very paradigm in looking at physics, and it may serve as a key to solving hard problems in the field such as a theory of everything, dark energy, and quantum gravity.

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[1] J. Bae, J.-W. Lee, J. Kim, and W.-Y. Hwang, Phys. Rev. A **78**, 022335 (2008).
[2] R. Landauer, IBM J. Res. Dev. **5**, 183 (1961).
[3] A. Zeilinger, Found. Phys. **29**, 631 (1999).
[4] C. Rovelli, Int. J. of Theor. Phys. (1996).
[5] J. D. Bekenstein, Phys. Rev. D **7**, 2333 (1973).
[6] E. P. Verlinde, arXiv:1001.0785 (2010).
[7] T. Padmanabhan, arXiv:0912.3165 (2009).
[8] L. Zhao, (2010), 1002.0488.
[9] R.-G. Cai, L.-M. Cao, and N. Ohta, (2010), 1002.1136.
[10] R.-G. Cai, L.-M. Cao, and N. Ohta, (2010), 1001.3470.
[11] Y. S. Myung, (2010), 1002.0871.
[12] Y.-X. Liu, Y.-Q. Wang, and S.-W. Wei, arXiv:1002.1062 (2010).
[13] Y. Tian and X. Wu, (2010), 1002.1275.
[14] A. Pesci, (2010), 1002.1257.
[15] M. Diego, (2010), arXiv:1002.1941.
[16] I. V. Vancea and M. A. Santos, (2010), arXiv:1002.2454.
[17] R. A. Konoplya, (2010), 1002.2818.
[18] H. Culetu, (2010), arXiv:1002.3876.
[19] Y. Zhao, (2010), arXiv:1002.4039.
[20] S. Ghosh, (2010), arXiv:1003.0285.
[21] J. Munkhammar, (2010), arXiv:1003.1262.
[22] X. Kuang, Y. Ling, and H. Zhang, (2010), arXiv:1003.0195.
[23] J.-W. Lee, J. Lee, and H.-C. Kim, JCAP08(2007)005 (2007), hep-th/0701199.
[24] J.-W. Lee, To be published in Foundations of Physics (2010), arXiv:1005.2739.
[25] A. G. Lisi, arXiv:physics/0605068 (2006).
[26] W. G. Unruh and N. Weiss, Phys. Rev. D**29**, 1656 (1984).
[27] J.-W. Lee, (2010), 1005.2739.
[28] L. C. B. Crispino, A. Higuchi, and G. E. A. Matsas, Rev. Mod. Phys. **80**, 787 (2008), 0710.5373.
[29] D. Derbes, American Journal of Physics **64**, 881 (1996).
[30] R. Banerjee and B. R. Majhi, (2010), 1003.2312.
[31] H.-C. Kim, J.-W. Lee, and J. Lee, JCAP **0808**, 035 (2008), 0804.2579.
[32] J.-W. Lee, H.-C. Kim, and J. Lee, (2010), 1001.5445.
[33] M. Li and Y. Wang, arXiv:1001.4466 (2010).
[34] S. Gao, (2010), arXiv:1002.2668.
[35] S. Hossenfelder, (2010), 1003.1015.
[36] J.-W. Lee, (2010), 1003.4464.
[37] T. Jacobson, Phys. Rev. Lett. **75**, 1260 (1995).
[38] T. Padmanabhan, Class. Quant. Grav. **21**, 4485 (2004), gr-qc/0308070.
[39] J.-W. Lee, H.-C. Kim, and J. Lee, (2010), arXiv:1002.4568.
[40] C. Gao, arXiv:1001.4585 (2010).
[41] H.-C. Kim, J.-W. Lee, and J. Lee, Mod. Phys. Lett. A **25**, 1581 (2007), 0709.3573.
[42] J.-W. Lee, *Entanglement and Gravity*, Talk given at the KIAS Workshop on Quantum Information Sciences Aug. 19, 2009, Seoul Korea (2009).
[43] J. W. Lee, arXiv:1109.3542 [hep-th].
[44] J. W. Lee, arXiv:1109.3542 [hep-th].
[45] G. 't Hooft, Determinism beneath quantum mechanics, 2002, quant-ph/0212095.
[46] Y. Zhang, Y.-g. Gong, and Z.-H. Zhu, arXiv:1001.4677 (2010).
[47] S.-W. Wei, Y.-X. Liu, and Y.-Q. Wang, arXiv:1001.5238 (2010).
[48] D. A. Easson, P. H. Frampton, and G. F. Smoot, (2010), arXiv:1002.4278.
[49] J.-W. Lee, H.-C. Kim, and J. Lee, Mod. Phys. Lett. **A25**, 257 (2010), arXiv:0803.1987.
[50] M. Li, Phys. Lett. B **603**, 1 (2004).
[51] Q.-G. Huang and M. Li, JCAP **0408**, 013 (2004), astro-ph/0404229.
[52] X. Zhang and F.-Q. Wu, arXiv:astro-ph/0701405 (2007).
[53] Q.-G. Huang and Y. Gong, JCAP **2004**, 006 (2004).
[54] E. Komatsu *et al.*, (2010), 1001.4538.
[55] J. Lee, H.-C. Kim, and J.-W. Lee, Phys. Lett. B **661**, 67 (2007), hep-th/0702121.